AC electrical conductivity

- at low frequencies, e.g. < 1 kHz, we may think of electrical circuits
  ↳ Household power grid opertates at 120 Hz!

- at higher frequencies we begin to think about EM waves & light

  AM radio: 300 – 3000 kHz

  Microwaves/ telcomm wireless : 1 – 10 GHz

  Light (optical) : 400 – 800 THz

- technically all the same from a physics perspective
- be thinking about "light" for this discussion

- Consider time-dependent $\bar{E}$ field:

$$\bar{E}(t) = \mathrm{Re}\left[\bar{E}(\omega)\exp(-i\omega t)\right]$$

Drude eqn of motion:

$$\frac{d\bar{p}}{dt} = \frac{-\bar{p}}{\tau} - e\bar{E}$$

- assume soln's take the form:

$$\bar{p}(t) = \mathrm{Re}\left[\bar{p}(\omega)\exp(-i\omega t)\right]$$

$$\therefore\ -i\omega\bar{p}(\omega) = \frac{-\bar{p}(\omega)}{\tau} - e\bar{E}(\omega)$$

$$\left(\frac{1}{\tau} - i\omega\right)\bar{p}(\omega) = -e\bar{E}(\omega)$$

Recall:

$$\bar{j} = \frac{-ne\bar{p}}{m}$$

$$\therefore\ \bar{j}(\omega) = \underbrace{\frac{ne^2\tau}{m}}\left(\frac{1}{1-i\omega\tau}\right)\bar{E}(\omega)$$

= DC Drude conductivity
$\equiv \sigma_0$

- takes the form:

$$\bar{j}(\omega) = \sigma(\omega)\bar{E}(\omega)$$

$$\boxed{\sigma(\omega) = \frac{\sigma_0}{1-i\omega\tau}} \qquad \boxed{\sigma_0 = \frac{ne^2\tau}{m}}$$

→ complex conductivity: $\sigma(\omega) = \sigma_1(\omega) + i\sigma_2(\omega)$

Sanity-check:

$$\sigma(\omega=0) = \frac{\sigma_0}{1-i(0)\tau} = \sigma_0$$

Good!

Note: we ignored the $\bar{B}$ component of the EM wave
↳ justified? see pg. 16 → $\bar{B}$ contribution ~ 10^{-10} of $\bar{E}$

Note #2: We assumed a time-varying field, but spacially uniform field.

OK so long as $\lambda >> \ell_0$
↳ valid down to UV - x rays
↳ $\bar{e}$ mean free path

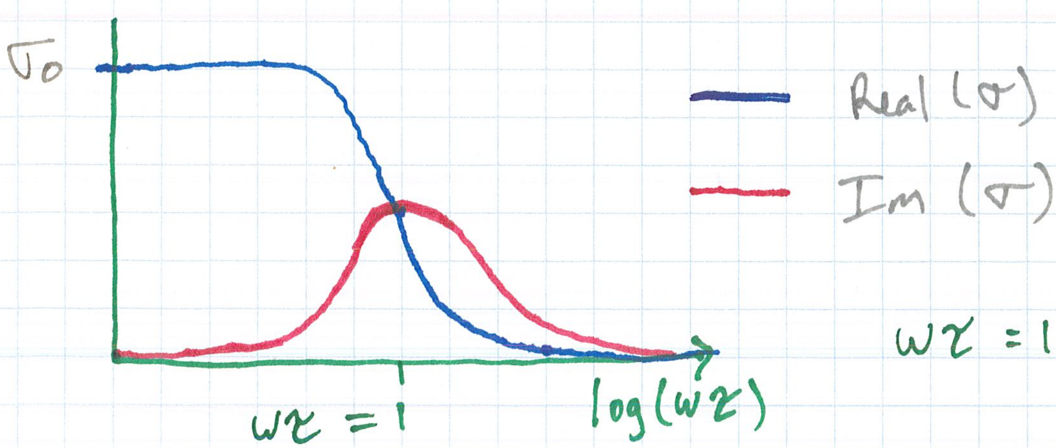
→ $\omega\tau << 1$: $\sigma(\omega) \approx \sigma_0$ ⇒ real

$\omega\tau >> 1$: $\sigma(\omega) \approx -\dfrac{\sigma_0}{i\omega\tau}$ ⇒ imaginary

We can write:

$$\sigma(\omega) = \sigma_1(\omega) + i\sigma_2(\omega)$$

$$= \frac{\sigma_0}{1+\omega^2\tau^2} + i\omega\tau \frac{\sigma_0}{1+\omega^2\tau^2}$$



$\omega\tau = 1 \approx$ THz (mid to far IR)

So what?

Maxwell's Eqn's (no net charge density)

$$\nabla \cdot \bar{E} = 0 \qquad \nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$$

$$\nabla \cdot \bar{B} = 0$$

$$\nabla \times \bar{B} = \mu_0 \bar{J} + \mu_0 \varepsilon_0 \frac{\partial \bar{E}}{\partial t}$$

$$\mu_0 \varepsilon_0 = \frac{1}{c^2}$$

Result: Problem Set #1

$$-\nabla^2 \bar{E} = \frac{\omega^2}{c^2}\left(1 + \frac{i\sigma(\omega)}{\varepsilon_0 \omega}\right)\bar{E}$$

EM wave eqn:

$$-\nabla^2 \bar{E} = \frac{\omega^2}{c^2}\,\epsilon(\omega)\,\bar{E}$$

$\therefore$ Complex dielectric constant:

$$\boxed{\epsilon(\omega) = 1 + \frac{i\sigma(\omega)}{\varepsilon_0 \omega}}$$

Sometimes we write:

$$\nabla^2 \bar{E} + k^2 \bar{E} = 0$$

where $k^2 = \frac{\omega^2}{c^2} \epsilon(\omega)$

$$\therefore \bar{E} = E_0 e^{-ikz}$$

for plane-wave travelling in z-dir.

k-vector is important ...

→ Look @ limits of $\epsilon(\omega)$:

$$\epsilon(\omega) = 1 + \frac{i \sigma(\omega)}{\varepsilon_0 \omega}$$

$$= 1 + \frac{i \sigma_0}{\varepsilon_0 \omega} \left(\frac{1}{1 - i\omega\tau} \right)$$

$$= 1 - \frac{\tau \sigma_0}{\varepsilon_0 (1 + \omega^2 \tau^2)} + \frac{i \sigma_0}{\varepsilon_0 \omega (1 + \omega^2 \tau^2)}$$

$\omega\tau \ll 1$:

$$\epsilon(\omega) \approx i \frac{\sigma_0}{\omega \varepsilon_0}$$

→ imaginery dielectric constant (parameter)

$$k^2 = i \frac{\omega^2}{c^2} \frac{\sigma_0}{\omega \varepsilon_0}$$

$$\therefore \quad k = \pm \frac{(1+i)}{\sqrt{2}} \sqrt{\frac{\sigma_0 \omega}{\varepsilon_0 c^2}}$$

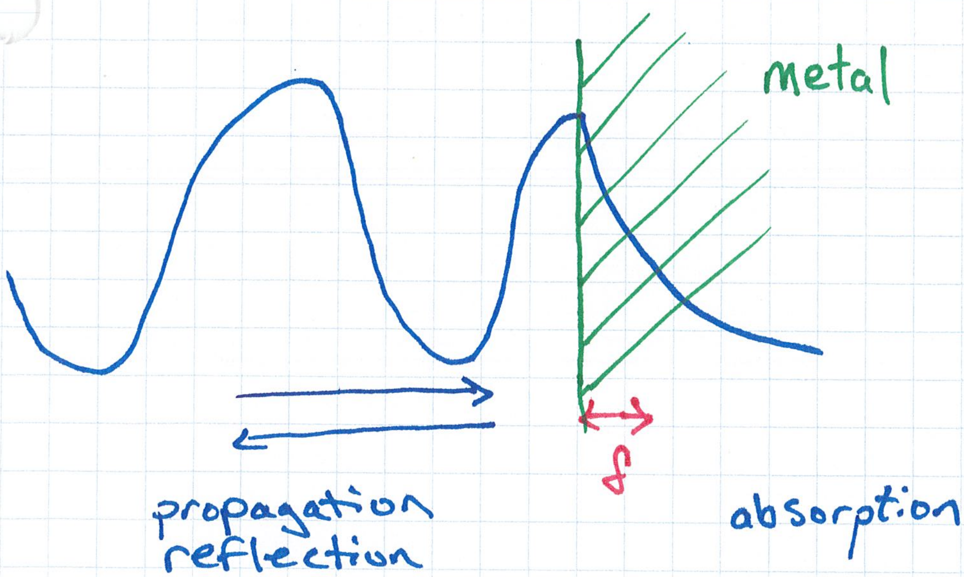
$$\sqrt{i} = \left(e^{i\pi/2}\right)^{1/2} = e^{i\pi/4} = \pm\left(\frac{1+i}{\sqrt{2}}\right)$$

Define δ "skin depth":

$$\delta \equiv \sqrt{\frac{c^2 \varepsilon_0}{\sigma_0 \omega}} = \sqrt{\frac{c^2 \varepsilon_0 m}{n e^2 \omega}}$$

$$\therefore \quad k = \pm \frac{(1+i)}{\sqrt{2}} \frac{1}{\delta}$$

δ is length scale of exp. decay into metal.



$\omega\tau \gg 1$: High frequency / short wavelength

$$\epsilon(\omega) \approx 1 - \frac{\sigma_0}{\omega^2 \tau \varepsilon_0} = 1 - \frac{ne^2}{\omega^2 m \varepsilon_0}$$

Define ω_p "plasma frequency":

$$\omega_p \equiv \sqrt{\frac{ne^2}{m\varepsilon_0}}$$

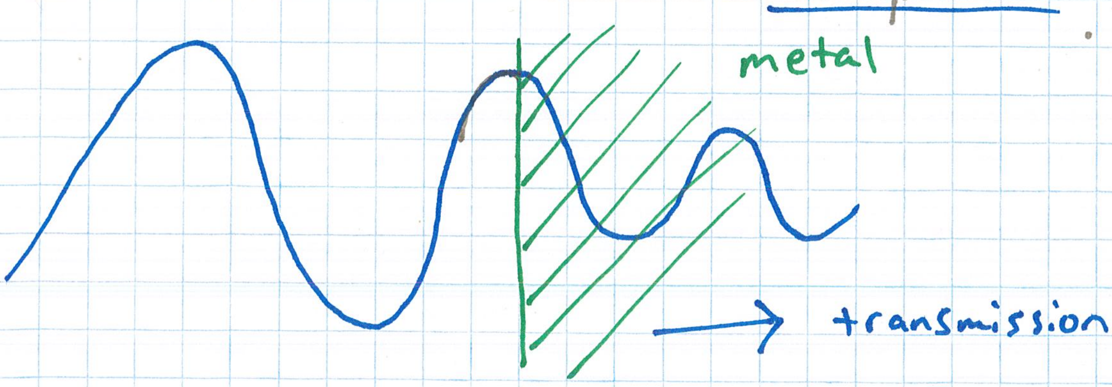
Note: $\omega_p = \frac{c}{\delta}$ → skin depth

$$\therefore \quad \epsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2}$$

$$k^2 = \frac{\omega^2}{c^2}\left(1 - \frac{\omega_p^2}{\omega^2}\right)$$

→ if $\omega > \omega_p$, k is real and we have a propagating wave in the metal...

i.e. the metal is transparent!



How high of freq. is ω_p?

$$\lambda_p = \frac{2\pi c}{\omega_p}$$ → plug in n for simple metals!

$\lambda_p \approx 100-400$ nm ⇒ UV ⇒ see A&M Table 1.5

→ Metals are generally reflective up to visible frequencies, then become transparent in the UV.

Frequencies below ω_p (reflection / abs regime)

→ the AC $\bar{E}$ field can induce oscillations in charge density @ the surface of the metal

→ called "plasma oscillations"

<u>OR</u> plasmons → quantized!

Note: this oscillation of charge is the mechanism for <u>reflection</u>!

Complex refractive index:

$$n' = n + iK = \sqrt{\epsilon(\omega)}$$

K → extinction coefficient

$\epsilon(\omega)$ → dielectric parameter

n ↳ index of refraction

Reflectance:

$$R = \frac{(n-1)^2 + K^2}{(n+1)^2 + K^2}$$

e.g. glass: $n \approx 1.5$, $K \approx 0$ $R \approx 4\%$

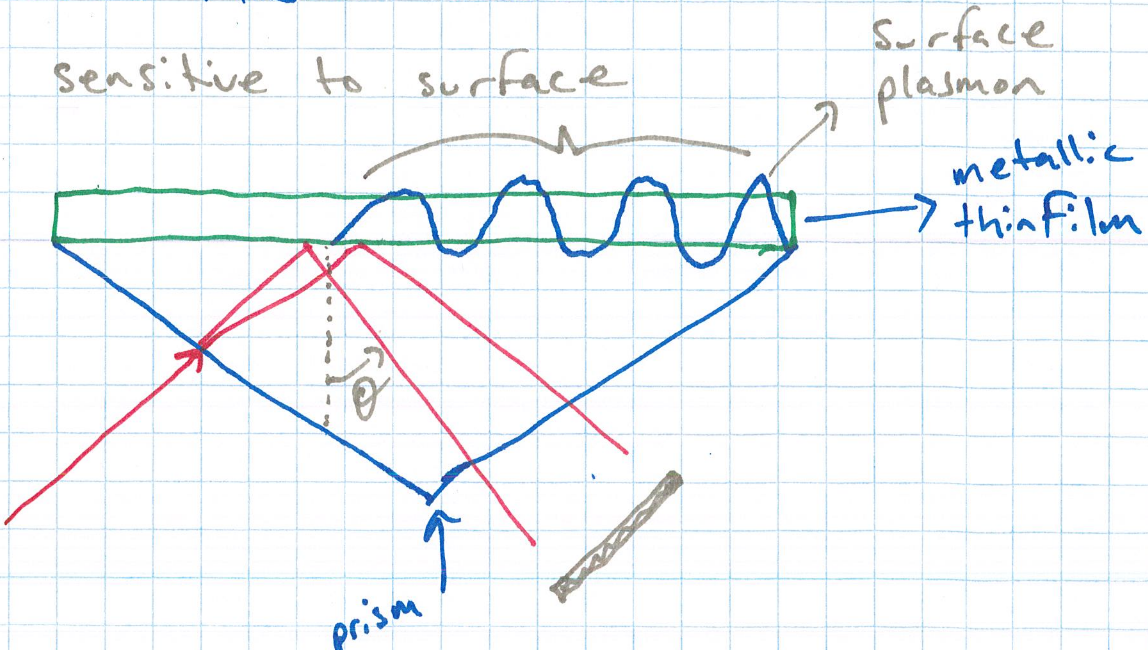
Ag (silver): $n \approx 10.1$, $K \approx 100$ $R \approx 99.6\%$
@ $14\mu m$

→ can compute n & K from $\epsilon(\omega)$, which we just showed.

Surface Plasmons: → plasma oscillations @ surface

Surface Plasmon Resonance

sensitive to surface



→ monitor SPR peak.

→ layer-by-layer growth

→ antibody-receptor binding